

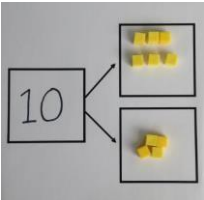

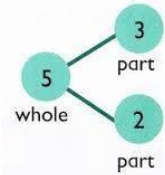

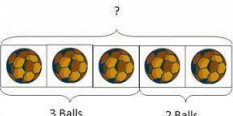

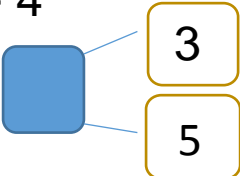

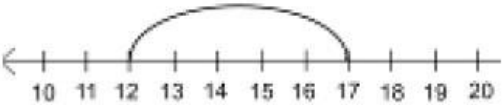

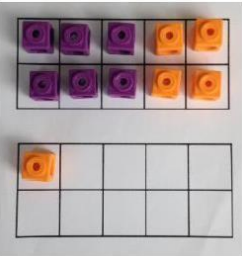

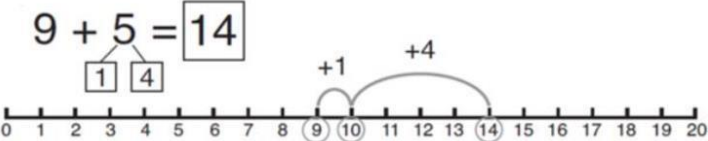
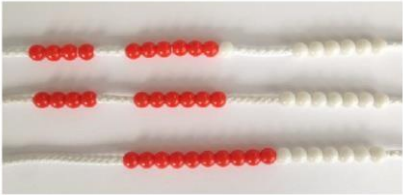
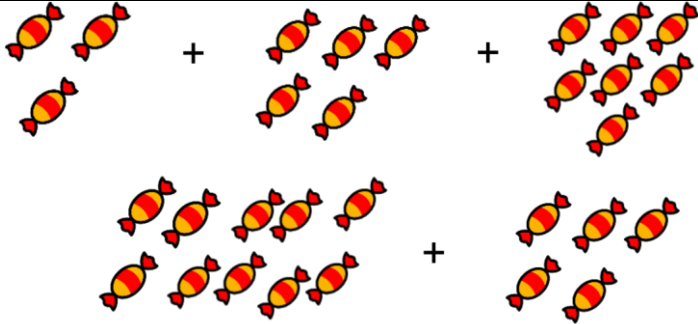
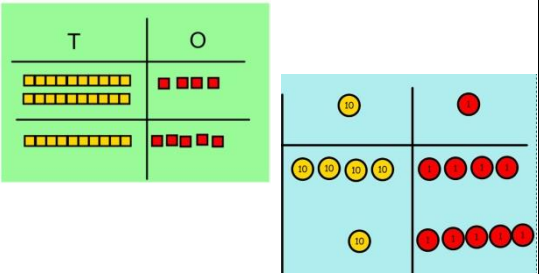
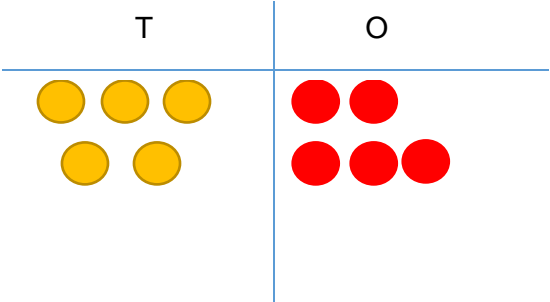


Progression in Calculations

Addition

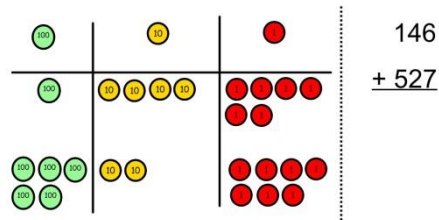
| Objective and Strategies | | Concrete | Pictorial | Abstract |
|--------------------------|---|--|--|---|
| Year One | Combining two parts to make a whole: part-whole model |     <div>Use cubes to add two numbers together as a group or in a bar.</div> |    <div>Use pictures to add two numbers together as a group or in a bar.</div>  | $4 + 3 = 7$ $10 = 6 + 4$  <div>Use the part-part whole diagram as shown above to move into the abstract.</div> |
| | |  <div>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</div> | $12 + 5 = 17$  <div>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</div> | $5 + 12 = 17$ <div>Place the larger number in your head and then count on using the smaller number to find your answer.</div> |

| | | | | |
|----------|-----------------------------------|--|--|--|
| Year One | <p>Regrouping to make 10.</p> |  <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p> |  <p>$3 + 9 =$</p> <p>$9 + 5 = 14$</p>  <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> | <p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p> |
| Year Two | <p>Adding three single digits</p> | <p>$4 + 7 + 6 = 17$</p> <p>Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p> |  <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p> | <p>$4 + 7 + 6 = 10 + 7$</p> <p>$= 17$</p> <p>Combine the two numbers that make 10 and then add on the remainder.</p> |

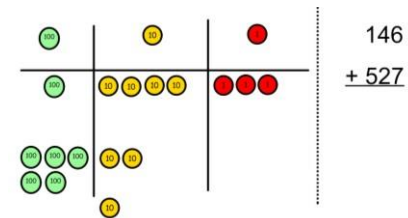
| | | | | |
|----------|-------------------------------------|---|--|--|
| Year Two | <p>Column method- no regrouping</p> | <p>$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p>  <p>The image shows two representations of the addition 24 + 15. On the left, a green place value chart with columns 'T' (Tens) and 'O' (Ones) contains yellow base 10 blocks: two tens rods and four ones units in the 'T' column, and one ten rod and five ones units in the 'O' column. On the right, a blue place value chart shows the same numbers using circular counters: two yellow tens counters and four yellow ones counters in the 'T' column, and one yellow tens counter and five red ones counters in the 'O' column.</p> | <p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p>  <p>A place value chart with columns 'T' (Tens) and 'O' (Ones) separated by a vertical line. The 'T' column contains five yellow circular counters (two in the top row, three in the bottom row). The 'O' column contains seven red circular counters (two in the top row, five in the bottom row).</p> | <p><u>Calculations</u></p> <p>$21 + 42 =$</p> $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$ |
|----------|-------------------------------------|---|--|--|

Column method-regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

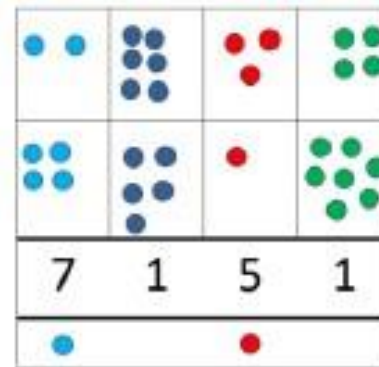


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

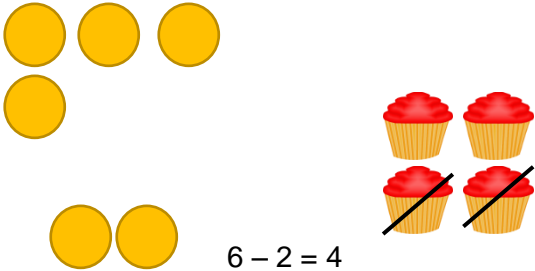
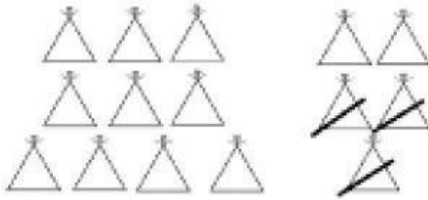
$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$



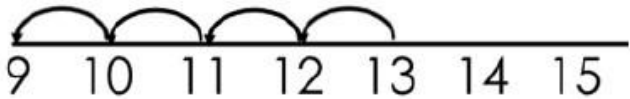
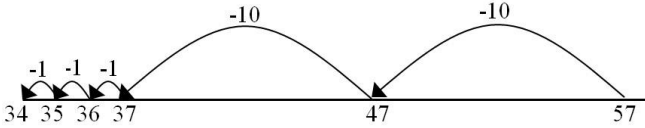
$$\begin{array}{r} £ 23.59 \\ + £ 7.55 \\ \hline £ 31.14 \end{array}$$

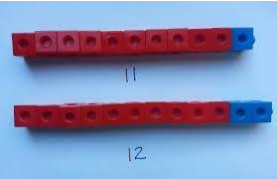
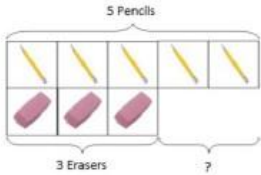
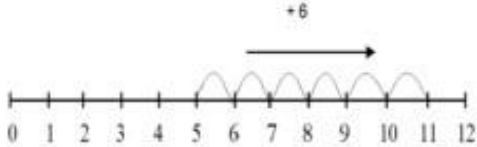
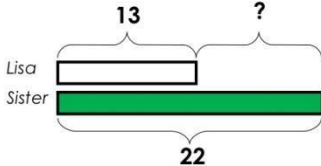
$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

| Year | One | Two | Three | Four | Five | Six |
|-----------------------------------|--|--|---|--|---|--|
| With jottings ... or in your head | Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$ | Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers | Add and subtract numbers mentally, including: <ul style="list-style-type: none"> * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds | Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why | Add and subtract numbers mentally with increasingly large numbers | Perform mental calculations, including with mixed operations and large numbers |
| Just know it! | Represent & use number bonds and related subtraction facts within 20 Add and subtract one-digit and two-digit numbers to 20, including zero | Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 | | | | |
| Addition guidance | Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10. | Adding three single digits. Column method – no regrouping. | Column method-regrouping. (up to 3 digits) | Column method-regrouping. (up to 4 digits) | Column method-regrouping. (with more than 4 digits) (Decimals- with the same number of decimal places) | Column method-regrouping. (Decimals- with different amounts of decimal places) |

Subtraction

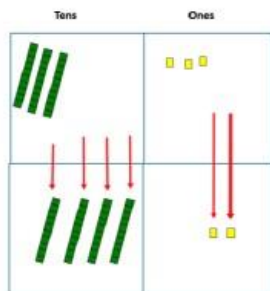
| Objective and Strategies | | Concrete | Pictorial | Abstract |
|--------------------------|------------------|---|--|---|
| Year One | Taking away ones | <p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p> | <p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p> | <p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p> |

| | | | | |
|--------------|----------------------|---|---|---|
| Year One/Two | <p>Counting back</p> | <p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p>  | <p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p> | <p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> |
|--------------|----------------------|---|---|---|

| | | | | |
|--------------|----------------------------|--|---|---|
| Year One/Two | <p>Find the difference</p> | <p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p> | <p>Count on to find the difference.</p>  <p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  | <p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p> |
|--------------|----------------------------|--|---|---|

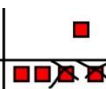
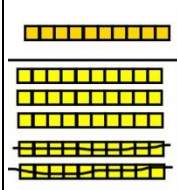
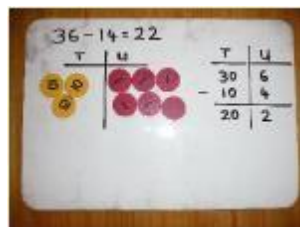
| | | | | |
|--------------|----------------------------------|--|--|--|
| Year One/Two | <p>Part Part Whole Model</p> | <div data-bbox="416 129 616 325"> </div> <p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p> | <p>Use a pictorial representation of objects to show the part whole model.</p> <div data-bbox="1077 201 1621 480"> </div> | <div data-bbox="1789 201 1998 384"> </div> <p>Move to using numbers within the part whole model.</p> |
| Year One/Two | <p>Make 10</p> | <p>$14 - 9 =$</p> <div data-bbox="405 528 920 703"> </div> <p>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p> | <div data-bbox="965 536 1659 639"> </div> <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p> | <p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p> |

Column method without regrouping



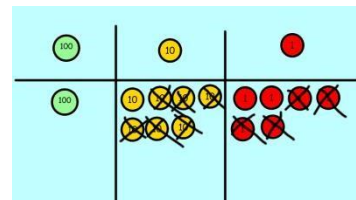
Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$



Calculations

$$176 - 64 =$$

$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$

working.

Draw the Base 10 or place value counters alongside the written calculation to help to show

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

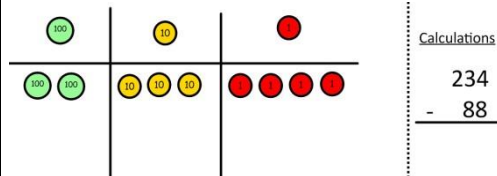
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

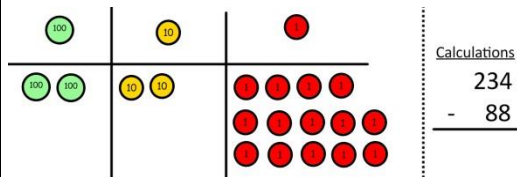
Make the larger number with the place value counters



Calculations

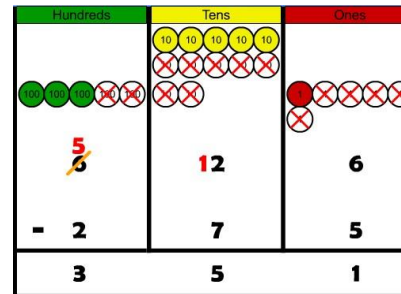
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

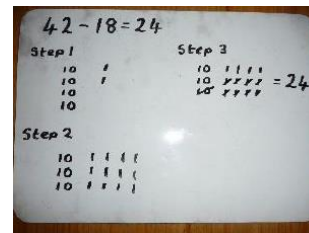


Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

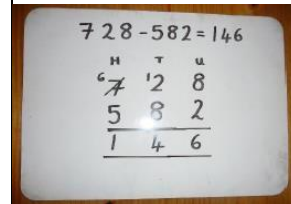


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

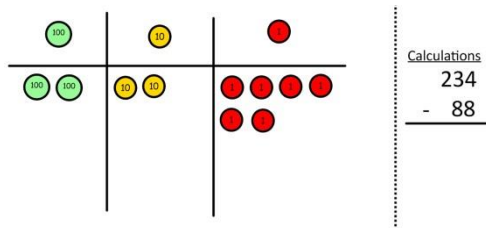


Children can start their formal written method by partitioning the number into clear place value columns.

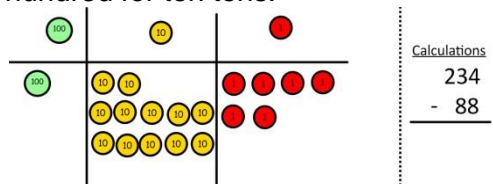


Moving forward the children use a more compact method.

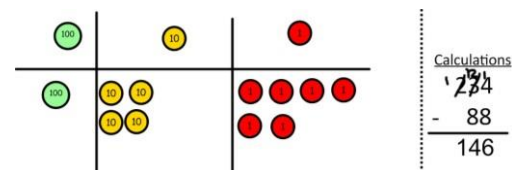
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction



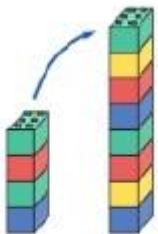

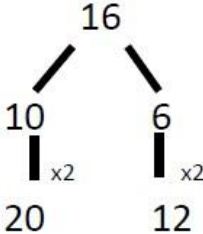
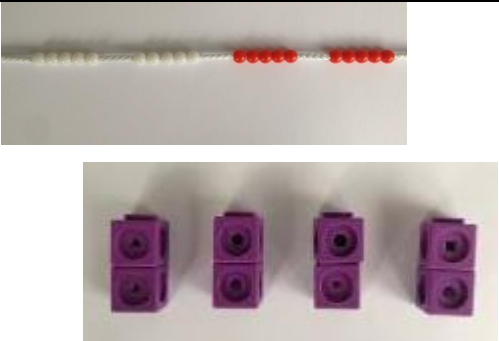
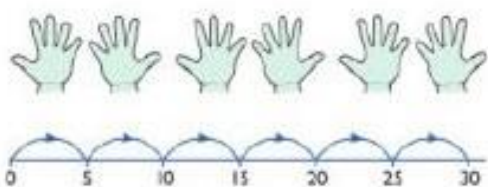
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

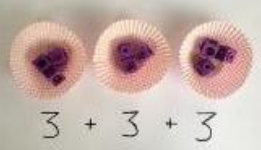



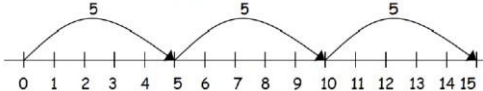





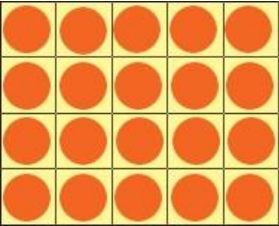
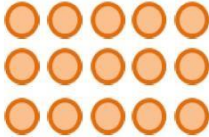
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \cancel{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

| Year | One | Two | Three | Four | Five | Six |
|-----------------------------------|--|--|---|--|--|---|
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| Just know it! | Represent and use number bonds and related subtraction facts within 20 Add and subtract one-digit and two-digit numbers to 20, including zero | Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 | | | | |
| Subtraction guidance | Taking away ones Counting back Find the difference Part whole model Make 10 | Counting back Find the difference Part whole model Make 10 Column method- no regrouping | Column method with regrouping. (up to 3 digits) | Column method with regrouping. (up to 4 digits) | Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places) | Column method with regrouping. (Decimals- with different amounts of decimal places) |

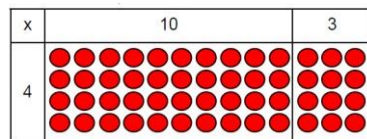
Multiplication

| Objective and Strategies | | Concrete | Pictorial | Abstract |
|--------------------------|-----------------------|---|--|--|
| Year One/Two | Doubling | <p>Use practical activities to show how to</p>  <p>double 4 is 8 $4 \times 2 = 8$</p> <p>double a number.</p> | <p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p>  |  <p>Partition a number and then double each part before recombining it back together.</p> |
| | Counting in multiples |  <p>Count in multiples supported by concrete objects in equal groups.</p> |  <p>Use a number line or pictures to continue support in counting in multiples.</p> | <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> |

| | | | | |
|---|---|--|--|--|
| Year Two | <p>Repeated addition</p> |    <div data-bbox="714 339 947 477"> <p>Use different objects to add equal groups.</p> </div> | <p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p>2 add 2 add 2 equals 6</p>  <p>$5 + 5 + 5 = 15$</p> | <p>Write addition sentences to describe objects and pictures.</p>  <p>$2 + 2 + 2 + 2 + 2 = 10$</p> |
| Year Two/Three (Year One with support) | <p>Arrays- showing commutative multiplication</p> | <p>Create arrays using counters/ cubes to show multiplication sentences.</p>   | <p>Draw arrays in different rotations to find commutative multiplication sentences.</p>  <p>$4 \times 2 = 8$</p> <p>$2 \times 4 = 8$</p>  <p>$2 \times 4 = 8$</p> <p>$4 \times 2 = 8$</p>  <p>Link arrays to area of rectangles.</p> | <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$</p> <p>$3 + 3 + 3 + 3 + 3 = 15$</p> <p>$5 \times 3 = 15$</p> <p>$3 \times 5 = 15$</p> |

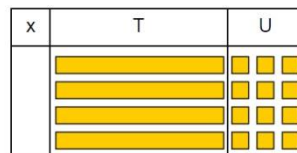
Grid Method

Show the link with arrays to first introduce the grid method.



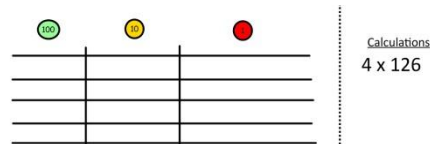
4 rows
of 10
4 rows
of 3

Move on to using Base 10 to move towards a more compact method.



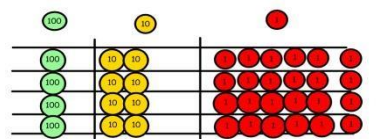
4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



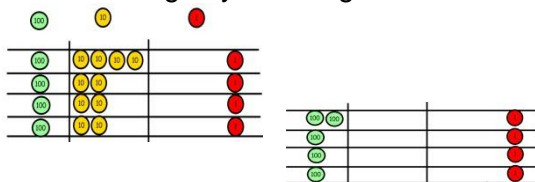
Calculations
4 x 126

Fill each row with 126.



Calculations
4 x 126

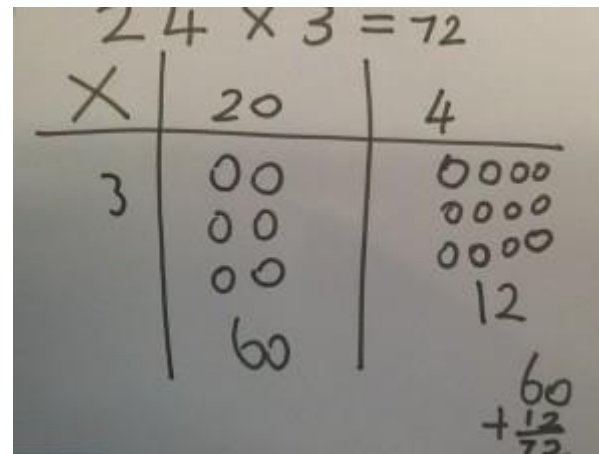
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

| | | |
|---|-----|----|
| x | 30 | 5 |
| 7 | 210 | 35 |

$$210 + 35 = 245$$

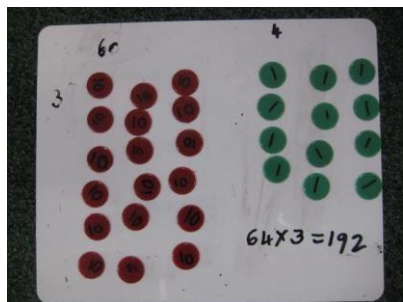
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

| | | |
|----|-----|----|
| | 10 | 8 |
| 10 | 100 | 80 |
| 3 | 30 | 24 |

| | | | | |
|----|-------|------|-----|----|
| x | 1000 | 300 | 40 | 2 |
| 10 | 10000 | 3000 | 400 | 20 |
| 8 | 8000 | 2400 | 320 | 16 |

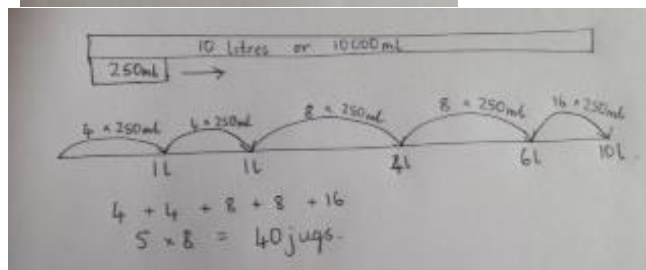
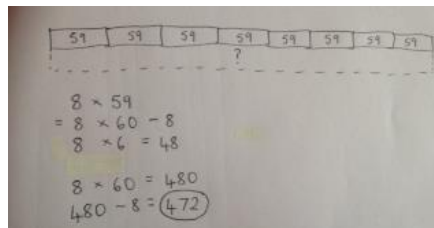
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

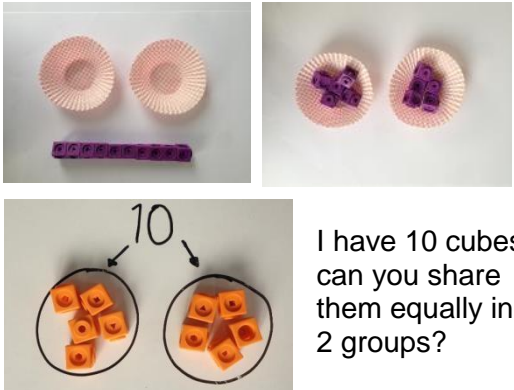
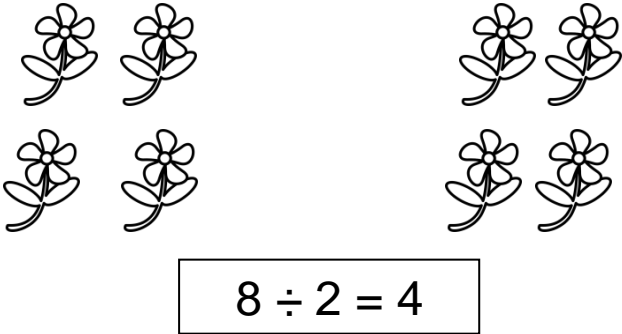
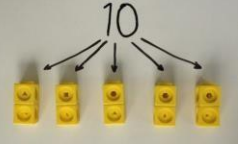
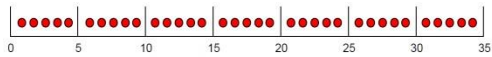
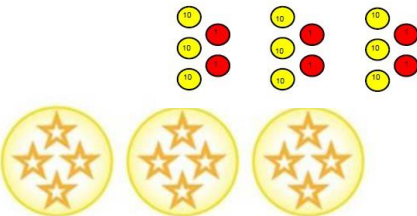
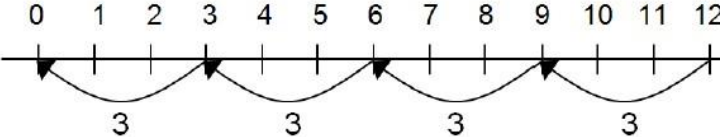
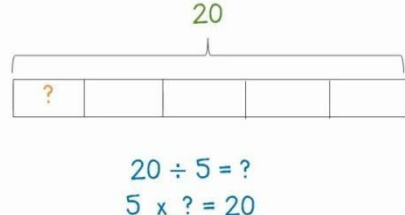
$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

This moves to the more compact method.

$$\begin{array}{r} 2 3 1 \\ 1342 \\ \times 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \\ 1 \end{array}$$

| Year | One | Two | Three | Four | Five | Six |
|-----------------------------------|---|--|---|---|---|--|
| With jottings ... or in your head | Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher | Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts | Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods | Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations | Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers establish whether a number up to 100 is prime | Perform mental calculations, including with mixed operations and large numbers |
| Just know it! | Count in multiples of twos, fives and tens | Recall and use \times and \div facts for the 2, 5 and 10 \times tables, including recognising odd and even numbers. | Recall and use \times and \div facts for the 3, 4 and 8 times tables. | Recall \times and \div facts for \times tables up to 12×12 . | Recall prime numbers up to 19 know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3) | |
| Multiplication guidance | Doubling Counting in multiples Arrays (with support) | Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication | Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method | Column multiplication (2 and 3 digit multiplied by 1 digit) | Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits) | Column multiplication (multi digit up to 4 digits by a 2-digit number) |

Division

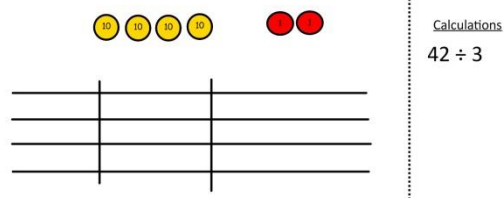
| Objective and Strategies | Concrete | Pictorial | Abstract |
|---|--|--|---|
| Year One Sharing objects into groups |  <p>I have 10 cubes, can you share them equally in 2 groups?</p> | <p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$ | <p>Share 9 buns between three people.</p> $9 \div 3 = 3$ |
| Year One/Two Division as grouping | <p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>   $96 \div 3 = 32$  | <p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$ | $28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p> |

| | | | | |
|---------------------|----------------------------------|---|---|---|
| Year Two/Three/Four | <p>Division within arrays</p> | <div data-bbox="416 134 736 341" data-label="Image"> </div> <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p> | <div data-bbox="999 165 1662 437" data-label="Image"> </div> <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p> | <p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p> |
| Year Two/Three/Four | <p>Division with a remainder</p> | <p>$14 \div 3 =$ Divide objects between groups and see how much is left over</p> <div data-bbox="412 724 931 1091" data-label="Image"> </div> | <p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p> <div data-bbox="967 628 1671 756" data-label="Figure"> </div> <p>Draw dots and group them to divide an amount and clearly show a remainder.</p> <div data-bbox="1061 916 1514 1011" data-label="Image"> </div> | <p>Complete written divisions and show the remainder using r.</p> <div data-bbox="1733 724 2069 788" data-label="Equation-Block"> $\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER} & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$ </div> |

Short division

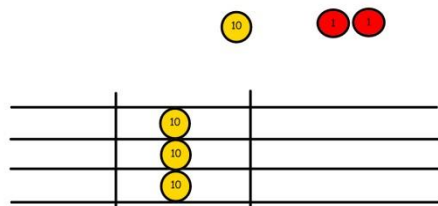


Use place value counters to divide using the bus stop method alongside

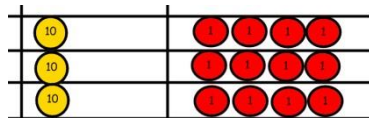


$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

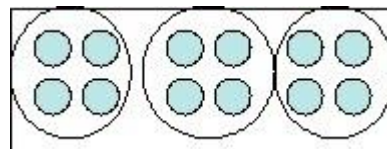


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$$

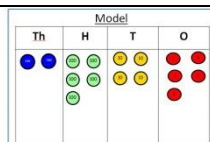
Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

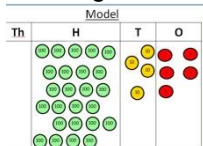
$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$

Long division



$2544 \div 12$
How many groups of 12 thousands do we have?
None

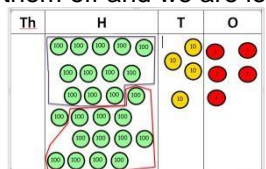
Exchange 2 thousand for 20 hundreds.



$$12 \overline{) 2544}$$

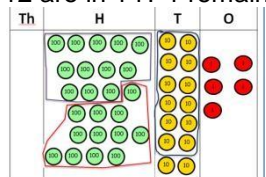
How many groups of 12 are in 25 hundreds?
2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



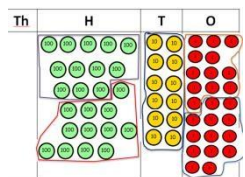
$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of



$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

12 are in 24? 2

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

$$\begin{array}{r} 0318r5 \\ 20 \overline{) 6365} \\ \underline{-60} \\ 36 \\ \underline{-30} \\ 65 \\ \underline{-60} \\ 5 \end{array}$$

| Year | One | Two | Three | Four | Five | Six |
|-----------------------------------|---|--|---|---|---|--|
| With jottings ... or in your head | Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher | Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts | Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods | Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations | Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 | Perform mental calculations, including with mixed operations and large numbers |
| Just know it! | Count in multiples of twos, fives and tens | Recall and use \times and \div facts for the 2, 5 and 10 \times tables, including recognising odd and even numbers. | Recall and use \times and \div facts for the 3, 4 and 8 times tables | Recall \times and \div facts for \times tables up to 12×12 . | Recall prime numbers up to 19 know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers | |
| Division guidance | Sharing objects into groups Division as grouping | Division as grouping Division within arrays | Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial) | Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial) | Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context) | Short division Long division (up to 4 digits by a 2-digit number- interpret remainders as whole numbers, fractions or round) |

Glossary of Terms

2-digit number – a number with 2 digits like 23, 45, 12 or 60

3-digit number – a number with 3 digits like 123, 542, 903 or 561

Addition facts – knowing that $1+1 = 2$ and $1+3 = 4$ and $2+5 = 7$. Normally we only talk about number facts with totals of 20 and under.

Array - An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

Bead String/Bar – a string with (usually 100) beads on, grouped by colour in tens. The bead string is a good bridge between a number track and a number line as it maintains the cardinality of the numbers whilst beginning to develop the concepts of counting ‘spaces’ rather than objects.

Bridging – when a calculation causes you to cross a ‘ten boundary’ or a ‘hundred boundary’ e.g. $85 + 18$ will bridge 100.

Compact vertical – the name of the recommended written method for addition whereby the numbers are added in columns, 1s first then 10s and so on. Where the total exceeds 10, the ten 1s are exchanged for a 10 and written below the answer line. Sometimes referred to as ‘carrying’.

Concrete apparatus – objects to help children count and calculate– these are most often cubes (multilink) but can be anything they can hold and move including Cuisenaire rods, Dienes rods (hundreds, tens and units blocks), straws, Numicon, Place Value counters and much more.

Count all – when you add by counting all the items/objects e.g. to add 11 and 5 you would count out 11, then count out 5, then put them together and count them all to get **16**.

Count on – when you add (or sometimes subtract) by counting onwards from a given number. E.g. to add 11 and 5 you would count on 5 from 11 i.e. 12, 13, 14, 15, **16**

Decimal number – a number with a decimal point e.g. 2.34 (said as two point three four)

Decomposition – the name of the recommended written method for subtraction whereby the smaller number is subtracted from the larger, 1s first then 10s and so on. Where the subtraction cannot be completed as the second number is larger than the first, a 10 is exchanged for ten 1s to facilitate this. This is the traditional ‘borrowing’ form of column method, which is different to the ‘payback’ method.

Dienes Rods (or Base 10) – this is a set of practical equipment that represents the numbers to help children with place value and calculation. The Dienes rods show 1s, 10s, 100s and 1000s as blocks of cubes that children can then combine. Dienes rods do not break up so the child has to ‘exchange’ them for smaller or larger blocks where necessary.

Difference – the gap between numbers that is found by subtraction e.g. $7-5$ can be read as ‘7 take away 5’ or as the ‘difference between 7 and 5’

Dividend – the number being divided in a calculation

Divisor – the smaller number in a division calculation.

Double – multiply a number by 2

Efficient Methods – the method(s) that will solve the calculation most rapidly and easily

Equals - is worth the same as (be careful not to emphasise the use of = to show the answer)

Exchanging – Swapping a '10' for ten '1s' or a '100' for ten '10s' or vice versa (used in addition and subtraction when 'moving' 'ten' or a 'hundred' from its column into the next column and splitting it up). Heavily relied upon for addition and subtraction of larger numbers. Skills in this can be built up practically with objects, then Dienes rods/base 10, then place value counters before relying on a solely written method.

Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column

Factor – a number that divides exactly into another number, without remainder

Grid method – a method for multiplying two numbers together involving partitioning and multiplying each piece separately.

Grouping – an approach to division where the dividend is split into groups of the size of the divisor and the number of groups created are then counted.

Half - a number, shape or quantity divided into 2 equal parts

Halve – divide a number by 2

Integer - a whole number (i.e. one with no decimal point)

Inverse – the opposite operation. For example, addition is the inverse of subtraction and multiplication is the inverse of division.

Known Multiplication Facts – times tables and other number facts that can be recalled quickly to support with larger or related calculations e.g. if you know 4×7 then you also know 40×70 , 4×0.7 etc.

Long Division – formal written of division where the remainders are calculated in writing each time (extended version of short division)

Long Multiplication – formal written method of column multiplication

Multiple - a number which is an exact product of another number i.e. a number which is in the times table of another number

Number bonds – 2 numbers that add together to make a given total, e.g. 8 and 2 bond to 10 or 73 and 27 bond to 100

Number line – a line either with numbers or without (a blank numberline).

The number line emphasises the continuous nature of numbers and the existence of ‘in-between’ numbers that are not whole. It is based around the gaps between numbers.

Children use this tool to help them count on or count back for addition or subtraction. As they get older, children will count in ‘jumps’ on a number line e.g. to add 142 to a number they may ‘jump’ 100 and then 40 and then 2. The number line is sometimes used in multiplication and division but can be time consuming.

Number track – a sequence of numbers, each inside its own square. It is a simplified version of the number line that emphasises the whole numbers.

Numicon – practical maths equipment that teaches children the names and values of numbers 1-10 initially but then helps them with early addition, subtraction, multiplication and division. Numicon is useful for showing the real value of a number practically.

One-Step Calculation – a calculation involving only one operation e.g. addition. Usually the child must decide what that operation is.

Partition – split up a larger number into parts, such as the hundreds, tens and units e.g. 342 can be partitioned into 300 and 40 and 2

Place Value – the value of a digit created by its position in a number e.g. 3 represents thirty in 234 but three thousand in 3567

Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add the hundreds together, then add the tens to that total, then add the units to that total

Remainder – a whole number left over after a division calculation

Repeated addition – repeatedly adding groups of the same size for multiplication

Scaling – an approach to multiplication whereby the number is ‘scaled up’ by a factor of the multiplier e.g. 4×3 means 4 scaled up by a factor of 3.

Sharing – an approach to division whereby the dividend is shared out into a given number of groups (like dealing cards)

Short Division - traditional method for division with a single digit divisor (this is a compact version of long division, sometimes called ‘bus stop’)

Significant digit – the digit in a number with the largest value e.g. in 34 the most significant digit is the 3, as it has a value of ‘30’ and the ‘4’ only has a value of ‘4’

Single digit – a number with only one digit. These are always less than 10.

Sum – the total of two or more numbers (it implies addition). Sum should not be used as a synonym for calculation.

Two-step calculation - a calculation where two different operations must be applied e.g. to find change in a shop you will usually have to add the individual prices and then subtract from the total amount. Usually the child has to decide what these two operations are and the order in which they should be applied.